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Turbulence-Model Transition Predictions

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I. Introduction

ALTHOUGH analytical tools for predicting transition from laminar to turbulent flow have improved significantly in recent years, transition remains one of the least understood phenomena of fluid mechanics. Classical linear stability analysis has been used extensively. While some insight into the transition phenomenon has attended this work, predictions often differ from experimental observations. Furthermore, linear analysis determines stability of a flow to infinitesimal disturbances only and is inapplicable when initial flow perturbations are of finite amplitude. Finally, in a linear stability analysis, complicating effects such as wall roughness, wall cooling, mass transfer, and freestream turbulence level and scale considerably increase the approach's mathematical complexity.

An alternative analytical method for transition prediction exists which in principle a) is applicable to arbitrary amplitude disturbances and b) in a simple and natural way can account for all of the complicating effects cited above. This method uses the Reynolds-averaged equations of motion subject to a set of closure hypotheses suitable for accurate computation through transition. Recent progress with phenomenological-turbulence-model equations indicates that this approach is sensible, i.e., that adequate closure approximations can indeed be determined. Using turbulence-model equations in which the Reynolds stresses depend upon flow history, Donaldson,[1] Jones and Launder,[2] and Wilcox[3] have shown that such sets of turbulence-model equations accurately predict abrupt transition from laminar to turbulent flow for constant-pressure boundary layers.

This Note presents recent results of turbulence-model transition research based on the Saffman turbulence model.[4] As discussed in Sec. II, the turbulence model has been modified to improve computational accuracy for transition predictions; then effects of freestream turbulence intensity and scale on model-predicted transition for an incompressible flat-plate boundary layer (FPBL) have been analyzed using the modified equations. The model also has been used to predict the minimum suction required to prevent FPBL transition and to make estimates of transition Reynolds number in channel and pipe flow.

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II. Formulation

The Saffman turbulence model assumes Reynolds stress, $\langle -u'v' \rangle$, is proportional to the mean velocity gradient, $\partial u/\partial y$, i.e.,

$$\langle -u'v' \rangle = \varepsilon\,\partial u/\partial y \tag{1}$$

where ε is the eddy viscosity. The eddy viscosity is postulated to be the ratio of the turbulent energy e and a turbulent pseudovorticity or dissipation rate ω, so that

$$\varepsilon = e/\omega \tag{2}$$

For incompressible boundary-layer flows, the "turbulence densities" e and ω satisfy the following nonlinear diffusion equations:

$$u\,\partial e/\partial x + v\,\partial e/\partial y = [\alpha^*|\partial u/\partial y| - \beta^*\omega]e + (\partial/\partial y)[(v+\sigma^*\varepsilon)\partial e/\partial y] \tag{3}$$

$$u\,\partial\omega^2/\partial x + v\,\partial\omega^2/\partial y = [\alpha|\partial u/\partial y| - \beta\omega]\omega^2 + (\partial/\partial y)[(v+\sigma\varepsilon)\partial\omega^2/\partial y] \tag{4}$$

where x and y denote distance parallel to and normal to the surface, u and v are velocity components in the x and y directions, and v is kinematic viscosity. The six parameters α, α^*, β, β^*, σ, σ^* are regarded as universal constants for fully developed turbulent flows, and their values have been established by general arguments based on well-documented experimental observations for such flows.[4]

The Saffman turbulence model has been incorporated in a boundary-layer program developed at the NASA Langley Research Center[5]; the modified program is known as EDDYBL. In using EDDYBL to make FPBL transition predictions, turbulent energy and pseudovorticity are held constant at the boundary-layer edge. Turbulent energy is set to zero throughout the boundary layer at a point near the plate leading edge. We then march in the streamwise direction and observe behavior of e. Some entrainment of e into the boundary layer initially occurs; however little or no turbulent-energy amplification occurs for a plate-length Reynolds number below a critical value Re_x^t, signifying existence of laminar flow. Then, when $Re_x \sim Re_x^t$, an abrupt increase in e is observed, followed by an asymptote to a value characteristic of fully developed turbulent flow. The transitional regime is readily identified as the range over which e increases from its initially low level to its much higher value in the turbulent regime. The transitional regime can also be identified from the numerical data by locating abrupt changes in quantities such as momentum thickness, shape factor, and skin friction.

Figure 1 shows computed skin friction c_f as a function of Re_x for an incompressible FPBL. The freestream value of e is $10^{-9}U^2$, where U is freestream velocity. As shown in the figure, the predicted transition begins at $Re_x = 4 \times 10^4$ and ends at

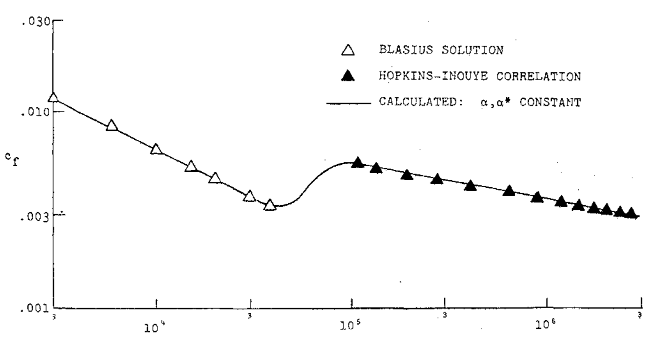

Fig. 1 Skin friction as a function of plate-length Reynolds number for an incompressible FPBL.

$Re_x = 10^5$. The predicted value of $Re_x{}^t$ of 4×10^4 is much lower than the measured $Re_x{}^t$ for very low freestream turbulence levels; Schubauer and Skramstad,[6] for example, indicate that $Re_x{}^t$ should be nearly 3×10^6 for $e \sim 10^{-7} U^2$.

Assuming α, α^*, β, β^*, σ, σ^* are independent of Reynolds number apparently leads to the inaccuracy. For low-Reynolds-number applications, these parameters depend upon the turbulent Reynolds number, $Re_T = e/\omega\nu$; other investigators[2] have, in fact, introduced a functional dependence of similar parameters upon Re_T. In the present study, we have found that transition is most sensitive to the values of α and α^*; decreasing α^* tends to delay transition while the ratio of α to α^* fixes the width of the transition zone. Therefore, to improve transition-prediction accuracy, we have assumed that

$$\alpha^* = \alpha_\infty{}^*[(1-\lambda)(1-Re_T/R_o)H(1-Re_T/R_o)] \tag{5}$$

$$\alpha/\alpha^* = \alpha_\infty/\alpha_\infty{}^* \tag{6}$$

where $H(x)$ is the Heaviside stepfunction and α_∞ and $\alpha_\infty{}^*$ are 0.2638 and 0.3, respectively, the values of α and α^* appropriate for fully turbulent flows.

The value of λ can be fixed by demanding that the Saffman-model neutral stability Reynolds number $\tilde{R}e_x$ be the same for a Blasius boundary layer as that predicted by linear stability theory. We define neutral stability as the condition where $\alpha^*|\partial u/\partial y| = \beta^*\omega$. Then, using the Blasius velocity profile and a computed ω-profile, we find

$$\tilde{R}e_x = 1000/\lambda^2 \tag{7}$$

with the least stable point being at approximately $y/\delta = 0.30$. Hence, using the accepted linear stability value of $\tilde{R}e_x$ of 9×10^5, we find $\lambda = 0.105$. A similar argument for selecting the value of R_o remains to be found. Numerical experimentation indicates that R_o should be about 0.10. However, this value should be regarded as tentative until further applications are made to determine its universality.

As a final note regarding the Saffman turbulence model and its application to transitional flows, we have found that algebraic transition predictions can be made which apparently are accurate to within a factor of two or three. Specifically, for many laminar flows over perfectly smooth surfaces, ω is given by $\omega = 20\nu/\beta y^2$ so that neutral stability occurs when $y^2|\partial u/\partial y|/\nu = 20/(\lambda\alpha_\infty{}^*\beta/\beta^*) = 317$. Also, transition will be complete when production of ω^2 exceeds ω^2 dissipation, i.e., when $y^2|\partial u/\partial y|/\nu = 20/(\lambda\alpha_\infty{}^*) = 722$. Therefore, an approximate transition criterion predicted by the Saffman model for incompressible flows is

$$317 < \max_y y^2|\partial u/\partial y|/\nu < 722 \tag{8}$$

Equation (8) resembles the Van Driest-Blumer[7] formulation.

III. Applications

Effects of freestream turbulence on FPBL transition have been analyzed using the modified Saffman turbulence model discussed in Sec. II. Results of the computations are presented in Fig. 2 where a subscript e denotes boundary-layer edge; transition Reynolds number, $Re_x{}^t$, is defined as the point where c_f is first observed to deviate from the laminar value by more than 0.5%. Turbulence intensity, T, is defined as $T = 100(2/3e_e)^{1/2}/U$. As shown, the model-predicted transition occurs at much higher values of Re_x than were obtained when α^* was assumed independent of Re_T. The effect of the freestream value of ω on predicted transition is also shown in Fig. 2; specifying a freestream boundary condition on ω is equivalent to fixing freestream turbulence scale. The effect of ω_e is most pronounced for high-intensity turbulence, with increasing ω_e tending to delay transition. Excellent agreement with all data shown is obtained when $\nu\omega_e/U^2 = 0.033$.

Effects of suction on FPBL transition have been analyzed to test the modified Saffman model's range of applicability. Computations have been performed for several suction rates to determine the minimum suction required to prevent transition. The indicated minimum volume coefficient ($C_Q = -v_o/U$ where v_o is suction velocity) required to prevent transition is $C_{Q\,\min} = 0.0017$. Experimental data are sparse and inconclusive for transitional FPBL flow with suction. Experiments by Simpson, Moffat, and Kays[8] for uniform suction indicate that $C_{Q\min}$ lies between 0.0024 and 0.0046, while Pfenninger's[9] experiments suggest that $C_{Q\min}$ is within the range 0.0010 to 0.0020. The computed $C_{Q\min}$ is hence within experimental data scatter. As a final note, the calculated result is far more accurate than the linear stability prediction[10] of $C_{Q\min} = 1.18 \times 10^{-4}$.

Fully developed channel and pipe flow are especially simple to analyze using the approximate transition criterion given in Eq. (8). Table 1 summarizes predictions for transition Reynolds number, R, based on average velocity and channel height/pipe diameter; experimentally measured R and the value of R predicted by linear stability analysis are also included in the table. Computed R agrees closely with measured R for both flows. As with the analysis of suction, Saffman-model predictions are much closer to corresponding measurements than are linear stability predictions, particularly for pipe flow.

Table 1 Transition Reynolds number predictions for fully developed channel and pipe flow

Flow	R, Present analysis	R, Measured	R, Linear stability analysis
Channel	1427–3249	1400	7085
Pipe	1070–2437	2300	∞

IV. Discussion

With one adjustable parameter at our disposal, namely R_o, we have made accurate transition predictions for a few incompressible boundary-layer flows. The effects of freestream turbulence and suction on an incompressible FPBL are well represented by the model. Also the analytical criterion given in Eq. (8) yields transition Reynolds numbers consistent with experimental measurements for channel and pipe flow. These results further indicate that accurate transition predictions can be made with phenomenological-turbulence-model equations. More definitive tests of the theory are warranted, particularly since turbulence-model transition prediction has the potential for

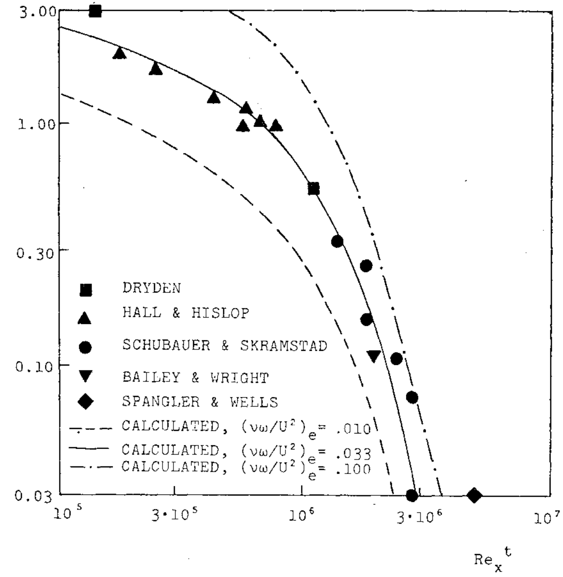

Fig. 2 Transition Reynolds number for incompressible FPBL flow as a function of freestream turbulence intensity T.

including, with little additional mathematical complication, effects such as wall roughness, surface heating/cooling, and compressibility.

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New Form for an Adaptive Observer

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Introduction

THE problem considered here is the estimation of the states and parameters of an nth order linear time-invariant plant where only the input and output can be observed. This development is an extension of the results of Lion[1] and Luders.[3] Lion considered parameter identification without state estimation. Luders[3] related the "observer"[2] state estimations structure to Lion's algorithm resulting in a state and parameter estimation algorithm. The adaptive observer formulation presented here has the advantage of greater "state variable filter"[1] separation than given in Ref. 3. In addition this formulation has greater design freedom in "state variable filter" selection.

Brief Development

It is assumed that the completely observable system can be described by an nth-order time-invariant vector differential equation. For the sake of simplicity, the new canonical form is derived only for the single-input single-output case. Nevertheless the extension of this canonical form to the multi-input case is straightforward.[4]

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Given

A stable stationary observable system transfer function with unknown parameters

$$G(s) = \sum_1^n \beta_i s^{i-1} \Big/ \left(s^n + \sum_1^n \alpha_i s^{i-1} \right) \tag{1}$$

find a convergent parameter and state estimator.

Solution

Restriction of Lion's[1] "state variable filter" to a simple form leads to a state estimate (observer) relationship. The transfer function Eq. (1) can be expressed more conveniently in terms of *known parameters* (λ) as follows:

$$G(s) = \frac{\left(b_n + \sum_1^{n-1} M_i b_i \right)}{\left(s + a_n + \lambda_n + \sum_1^{n-1} M_i a_i \right)} \tag{2}$$

where

$$M_i \triangleq \prod_{j=i}^{n-1} \frac{1}{(s+\lambda_j)}$$

In expression (2) the (a, b) are now the parameters to be identified. The transformation relating (a, b) to (α, β) involving (λ) can be derived easily by equating coefficients of like powers of s. Note that if all $\lambda_i = 0$ then $(a, b) = (\alpha, \beta)$. The form of Eq. (2) is motivated by state estimate convergence requirement.

The new canonic form is as follows:

$$\dot{w} = \bar{\Lambda} w \tag{3}$$

$$\dot{v} = \Lambda v + hb^T w \tag{4}$$

where

$$\bar{\Lambda} = \begin{pmatrix} -\lambda_1 & 1 & & 0 \\ & \ddots & \ddots & \\ 0 & & -\lambda_{n-1} & 1 \\ 0 & & & 0 \end{pmatrix}$$

$$\Lambda = \begin{pmatrix} -\lambda_1 & 1 & & 0 \\ & \ddots & \ddots & \\ 0 & & -\lambda_{n-1} & 1 \\ \hdashline -a_1 & & -a_{n-1} & -(\lambda_n + a_n) \end{pmatrix}$$

where $b^T = (b_1, \ldots, b_n)$; $h^T = (0, \ldots, 0, 1)$; $v_n = y =$ system output; and $w_n = u =$ system input.

Consider Eq. (4); treat w as a system input. Then the state "observer"[2] equation is given by Eq. (5):

$$\dot{\hat{v}} = \Lambda \hat{v} + hb^T w + k(y - \hat{y}) \tag{5}$$

Next select the "adaptive observer gain"†

$$k^T = (0, \ldots, 0, 1, -a_n) \tag{6}$$

and substituting $(\hat{a}, \hat{b}, \hat{w})$ for (a, b, w) in Eq. (5) we get the identifier Eq. (7).

$$\mathring{v} = \Lambda^* \begin{pmatrix} \hat{v}_1 \\ \vdots \\ \hat{v}_{n-1} \\ y \end{pmatrix} + h[\hat{b}^T \hat{w} - \lambda_n \hat{y}] \tag{7}$$

where

$$\Lambda^* = \Lambda + hh^T \lambda_n$$

† A minimal stationary (nonadaptive) observer results if we select $k^T = (0, \ldots, 0, 1, -\lambda_n)$.